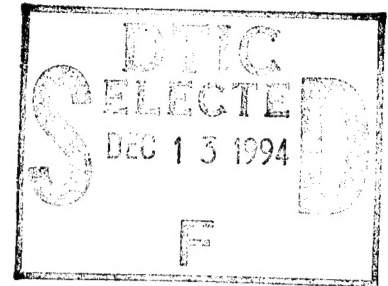NPS-ME-94-006

# NAVAL POSTGRADUATE SCHOOL
# MONTEREY, CALIFORNIA

## EFFECT OF SURFACE COATING ON ONE-DIMENSIONAL SYSTEM SUBJECTED TO UNIT STEP PRESSURE WAVE

by

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Prepared for:  Defense Nuclear Agency        Naval Postgraduate School
Alexandria, VA  20311            Monterey, CA  93943

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**Naval Postgraduate School**
**Monterey, California**


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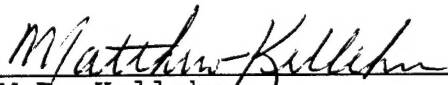

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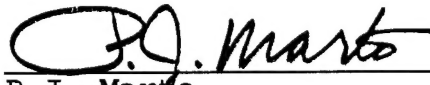Associate Professor of Mechanical Engineering


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Both the stress and nodal velocity of the structure increase with a decreasing coating stiffness regardless of the coating type or bounding medium. This phenomenon indicates that the coating stiffness governs the degree of strain energy release from the structure to the medium. A softer coating appears to trap this excess energy increasing the stress in the structure.

In all cases studied, the stiffer coating reduced the dynamic response of the structure when compared to the homogeneous system. A rubber shear modulus of approximately 6000 psi and greater ensured a favorable dynamic response for a coated aluminum structure enacted upon by a step pressure wave travelling in either air or water. The threshold value may vary depending upon the geometry and material properties of both the coating and the structure.

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# I. INTRODUCTION

Research at the Naval Postgraduate School continues in an effort to understand the dynamic response of coated structures subjected to shock waves. Past work focused on the comparison of numerical modeling to physical testing in an attempt to save on both time and expenses in performing research. Cylindrical models in an underwater environment subjected to both near and far field explosions have been tested with great success. Nelson, Shin, and Kwon [Ref. 1], Fox, Kwon, and Shin [Ref. 2] and Chisum [Ref. 3] have demonstrated that the coupled computer code of the finite element method and the boundary element method closely approximate simple experimental analyses. Hence this research asserts that limited parametric studies can be conducted without needing to construct and test physical models.

Kwon, Bergersen, and Shin [Ref. 4] studied the effects of surface coatings on metal cylinders in an underwater explosive environment. Under certain impact conditions, surface coatings appear to concentrate shock energy within the structure for longer time periods. This energy concentration manifests itself in higher stress and strain magnitudes in the metal cylinder. This is the result of trapping the shock wave energy and preventing its release into the surrounding water medium.

The amount of energy retained by the cylinder is greatly affected by both the thickness and shear modulus of the coating. In general, the resultant stress, strain, and deformation decrease with an increase in coating thickness and shear modulus. Both these parameters are used to categorize the stiffness of a material, which most likely determines the degree of energy transfer between a structure and a medium. Therefore, a threshold value for coating stiffness may be determined for a particular application. Above this theoretical value, a favorable dynamic response of a coated cylinder to an underwater explosion will occur; below this

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value, an adverse dynamic response results. An adverse response may entail increased strains and internal energy causing plastic deformation and failure of the structure.

Dissipation of energy into the surrounding medium is a critical factor to a structure's behavior in response to an explosion. The analysis of shock wave propagation and its effect on deformation is difficult due to the complexity of the interaction between the medium, coating, and structure.

The United States Navy has been experimenting with submarine coatings for decades. Hull coatings have been predominantly strategic in nature. For example, the rubber anechoic coating has been used as an anti-submarine warfare (ASW) tool to reduce acoustic energy reflected by the hull.

Despite the advantage provided in ASW, anechoic coatings may contribute to the adverse effects of a close-range underwater explosion. Previous studies have shown that coated cylinders have sustained greater shock damage than uncoated cylinders under identical testing conditions. The coating has prevented shock wave energy release to the surrounding water medium. This energy contributes to the elastoplastic deformation of the metal. When applied to a submarine, such a response has disastrous results for shock blast survivability of the vessel's crew and equipment.

In order to develop a coating with both ASW advantages and shock wave survivability, the effects of the coating on the structure need to be studied in greater detail. The first step involves examining the response of a simple system to a shock front to gain a thorough understanding of the medium-coating-structure interaction. Such research will provide insight into the physics which helps us to understand deformation and ultimate failure of the structure. A parametric, numerical study is performed on a simple, one-dimensional, aluminum structure to examine the interaction of two media and associated stress wave physics.

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## II. NUMERICAL ANALYSIS

In order to study the effects of shock waves on coated structures, a finite element model is developed. The premise of this study is to garner a basic understanding of a shock pulse impact and propagation through a coated structure.

The public-domain program used to develop the coated structure system is VEC/DYNA3D, an explicit finite element code designed by Livermore Software Technology Corporation [Ref. 5]. This particular code has been utilized quite extensively at the Naval Postgraduate School for evaluating the dynamic response of structures subjected to underwater explosions. VEC/DYNA3D provides a wide assortment of material types, equations of state, and loading conditions.

The pre-processor, LS-INGRID, is used to generate the actual finite element mesh [Ref. 6]. Interfacing with VEC/DYNA3D, INGRID constructs the model with respect to desired geometries, boundary conditions, planes of symmetry, material and element types, and external forces.

The VEC/DYNA3D calculations and outputs are reviewed in LS-TAURUS, an interactive post-processor [Ref. 7]. TAURUS displays the element, node, and material time history plots and other germane dynamic response characteristics.

For water-bounded systems, the finite element method provided by VEC/DYNA3D is used to model the structure and coating, but the water is modelled using a boundary element method code. Specifically, the boundary element method employed is the Underwater Shock Analysis (USA) code with the Cavitating Fluid Analyzer (CFA) upgrade designed by Dr. John A. DeRuntz, Jr. of Unique Software Applications [Ref. 8]. The doubly asymptotic approximation (DAA) developed by Dr. Thomas L. Geers of the University of Colorado provides the interaction between the acoustic water medium and the finite element model [Ref. 9]. The DAA reduces the number of elements needed to simulate the water medium.

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# III. MODEL DESCRIPTION

## A.   MATERIAL DESCRIPTION

An aluminum structure will be subjected to a step pressure wave not potent enough to cause plastic deformation. The material properties of the type of aluminum selected, 6061-T6, are given in Table 1. These values are standard handbook values for aluminum.

**Table 1. 6061-T6 Aluminum Properties**

| Parameter | Property/Aspect | Quantity |
|---|---|---|
| Density | $\rho$ | 5.447 slugs/ft$^3$ |
| Poisson's Ratio | $\upsilon$ | 0.33 |
| Young's Modulus | E | 1.08 x 10$^7$ psi |
| Yield Stress | $\sigma_y$ | 4.0 x 10$^4$ psi |
| Speed of sound | $c_o$ | 16,389 ft/sec |

The characteristics of the viscoelastic rubber coating is based on the Mooney-Rivlin compression model [Refs. 5,6]. This approach is suitable to the analysis of viscoelastic material deformation using general strain energy density. Mooney developed a new approach to study the deformation of soft material such as rubber or foam [Ref. 10]. He stated that the strain energy density function, W, is a function of the principal stretches (1 + principal extensions) of the material, $\eta$, the shear modulus, G, and a modulus expressing the asymmetry of reciprocal deformation, H. The variable H is a measure of the material's ability to store energy when compressed as opposed when it is stretched:

$$W = \frac{G}{4} \sum_{i=1}^{3} \left( \eta_i - \frac{1}{\eta_i} \right)^2 + \frac{H}{4} \sum_{i=1}^{3} \left( \eta_i^2 - \frac{1}{\eta_i^2} \right) \qquad (1)$$

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In order to lend versatility to this strain energy density theory and deformation under load, Mooney defined a new parameter which he termed the coefficient of asymmetry, $\alpha$:

$$\alpha = \frac{H}{G} \tag{2}$$

Values for the variables G and $\alpha$ were determined experimentally for tread stock rubber. Experimental data conducted with rubber undergoing up to 400% elongation and 50% compression correlated well with the analysis results. Thus the deformation of the rubber is characterized by the shear modulus and the coefficient of asymmetry.

The compressible Mooney-Rivlin rubber is implemented into VEC/DYNA3D to formulate the finite element model for the viscoelastic coating. The code requires two input constants, A and B. These two values are determined by the rubber shear modulus and coefficient of asymmetry as follows:

$$A = \frac{G}{4}(1 + \alpha) \tag{3}$$

$$B = \frac{G}{4}(1 - \alpha) \tag{4}$$

Tread stock rubber has the properties listed in Table 2.

**Table 2. Tread Stock Rubber Properties**

| Parameter | Property/Aspect | Quantity |
|---|---|---|
| Density | $\rho$ | 1.908 slugs/ft$^3$ |
| Shear Modulus | G | 95.8 psi |
| Speed of sound | $c_o$ | 100 ft/sec |
| Asymmetry coeff. | $\alpha$ | 0.223 |

## B.    GEOMETRIC DESCRIPTION

The model used for this study is a simple, one-dimensional system consisting of a coated structure subjected to a unit step pressure wave (Figure 1). The structure material is 6061-T6 aluminum, a widely-applied metal with excellent elastic properties; the coating is a elastic-plastic or viscoelastic material. One end of the system interfaces with either air or water and is free to displace; the other end is fixed. The step wave impacts the free end and propagates through both the coating and the structure.

This parametric study analyzes a finite element model consisting of 8-node hexahedral solid elements.  The major axis is in the x-direction. The system is bounded by the xy and xz symmetry planes. The structure and coating consist of 52 solid elements apiece; the overall system is composed of 420 nodes (Figure 2).

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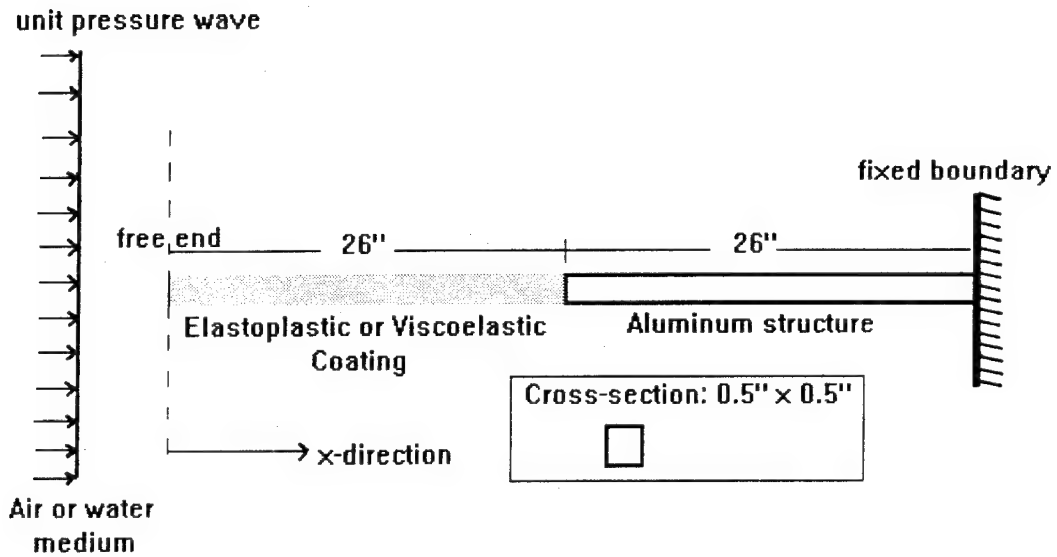
**Figure 1. One-dimensional system with symmetrical boundaries on all sides subjected to unit step pressure wave**



**Figure 2. Finite element model of the one-dimensional system**

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## IV. ANALYSIS OF ONE-DIMENSIONAL MODEL

### A.  FREE END BOUNDED BY AIR

#### 1. Elastic Coating

Kolsky [Ref. 11] asserts that stress wave propagation can be defined with the equations of motion expressed in terms of particle displacement. The three-dimensional displacement components, $u$, $v$, and $w$ in the $x$, $y$, and $z$ directions, respectively, satisfy the following equations:

$$\rho \frac{\partial^2 u}{\partial t^2} = (\lambda + \mu) \frac{\partial \Delta}{\partial x} + \mu \nabla^2 u \tag{5}$$

$$\rho \frac{\partial^2 v}{\partial t^2} = (\lambda + \mu) \frac{\partial \Delta}{\partial y} + \mu \nabla^2 v \tag{6}$$

$$\rho \frac{\partial^2 w}{\partial t^2} = (\lambda + \mu) \frac{\partial \Delta}{\partial z} + \mu \nabla^2 w \tag{7}$$

where:  $\rho$ = density of the solid containing the stress wave

$\Delta$ = dilatation, given by the following expression:

$$\Delta = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \tag{8}$$

$\lambda$ = Lame's constant, which is equal to

$$\lambda = k + \frac{2\mu}{3} \tag{9}$$

$k$ = bulk modulus of the structure

$\mu$ = material shear modulus

$\nabla^2$ = Laplace operator

Considering only one-dimensional displacement in the x-direction, the rest of this discussion will pertain to equation (5) only. The solution to this equation for an extended medium corresponds to both dilatational and distortional waves. Dilatational wave propagation is parallel to stress wave motion while distortional waves are

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perpendicular to this motion. Due to the one-dimensional restriction placed upon the model elements, only longitudinal vibrations will be retained. Therefore, displacement will take the form of alternating element contraction and extension with no lateral displacement along the main axis of the model.

### a. Homogeneous System

First consider a point just on the structure side of the interface of a model with the coating and structure of the same material. The common material is aluminum, a metal with good elastic behavior. In other words, the entire system is a homogeneous material. The ratio of the coating stiffness, $E_c$, to the structure stiffness, $E_s$, is unity. The term system will be used to describe the coating and structure as one integral unit. The free end of the system exposed to the air medium is subjected to an unit step pressure wave. The incident pressure wave will travel the length of the system without dispersion at rate $c_o$, the velocity of stress wave propagation. The compression wave will propagate through the uniform material directly to the fixed end of the structure (Figure 3a). There is no reflected wave at the interface between the coating and the structure because the characteristic impedance, $\rho c_o$, between the coating and structure is identical.

The pressure wave will produce varying degrees of displacement as it is transmitted through the system. The nodal displacement will be the largest at the free end and will decrease towards the fixed boundary. If the displacement created by the incident wave is expressed as:

$$u_1 = F(c_o t - x) \tag{10}$$

and the displacement created by a reflected pulse is given by:

$$u_2 = f(c_o t + x) \tag{11}$$

then from these above equations, the total displacement is:

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$$u_1 + u_2 = F(c_o t - x) + f(c_o t + x) \tag{12}$$

When the pressure pulse is reflected from a fixed surface, the boundary condition is one of zero-displacement. Due to this boundary condition, the above equations can be simplified to:

$$f(c_o t + x_o) = -F(c_o t - x_o) \tag{13}$$

where $x_o$ is the coordinate value at the fixed boundary. Thus the particle displacement behind the reflected wave, $u_2$, is equal and opposite to the particle displacement behind the incident wave. The pressure wave is completely reflected at a perfectly rigid or fixed boundary only both the direction of displacement and propagation are reversed. In other words, the stresses produced by the step wave are additive at the fixed end and the resultant stress is double the value of stress created by the incident wave.

The reflected pressure wave travels along the length of the system to the free end, the point of origin (Figure 3b). When the wave reaches the free end, it will be again reflected. However, the boundary condition here is one of no stress normal to the end face of the system. The characteristic impedance of the air is negligible in comparison to the coating. The stress produced by the two waves in the direction of propagation is given as follows:

$$E \frac{\partial u_1}{\partial x} \quad together\ with \quad E \frac{\partial u_2}{\partial x} \tag{14}$$

The cumulative stress at the free end is given by the following equation:

$$E \left( \frac{\partial u_1}{\partial x} + \frac{\partial u_2}{\partial x} \right) = E[-F'(c_o t - x) + f'(c_o t + x)] \tag{15}$$

If the free end is stress-free, then the above equation is simplified to:

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$$-F'(c_ot-x) + f'(c_ot+x) = 0 \qquad \textbf{(16)}$$

and the compressive wave is reflected as a like tensile wave.

The tensile wave relieves the additional compressive stress produced by the propagation of the reflected compressive wave (Figure 3c). In other words, it serves to undo some of the compressive stress caused by the passage of two compressive waves across the system. When this tensile wave reaches the fixed boundary, it is reflected as a tensile wave of equal magnitude (Figure 3d). The returning tensile wave undoes the remaining compressive stress giving the structure a zero-stress state. This cycle is identically repeated throughout the duration of the pressure wave. Figure 4 provides a summary of the events described above at a point on the structure.

### b. *Coating Less Stiff than Structure*

The previous discussion dealt with a homogeneous coating-structure system. In reality, there will be a difference in the characteristic impedance between the coating and the underlying structure due to the use of dissimilar materials. For example, consider the case where the coating maintains all the material properties of aluminum only the stiffness is reduced by a factor of 10 (i.e., $E_c/E_s = 0.1$). This rather fictitious material is used to understand the wave phenomenon for a system having two different characteristic impedances.

When the system is now acted upon by an external pressure wave, the resulting compressive stress wave now will first interact at the interface between the coating and the structure. The wave strikes this interface and is reflected back into the coating material as a compressive wave of smaller magnitude than the incident wave.

In order to maintain continuity at the interface, a net compressive wave of the same magnitude as both the incident and the reflected wave is transmitted into the structure

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air

C ———————→

Aluminum coating        Aluminum structure

free end        [3a]  initial compressive wave, C        fixed end

C' ←————————

free end        [3b]  reflected compression wave, C'        fixed end

T ———————→

free end        [3c]  reflected tensile wave, T        fixed end

T' ←————————

free end        [3d]  reflected tensile wave, T'        fixed end
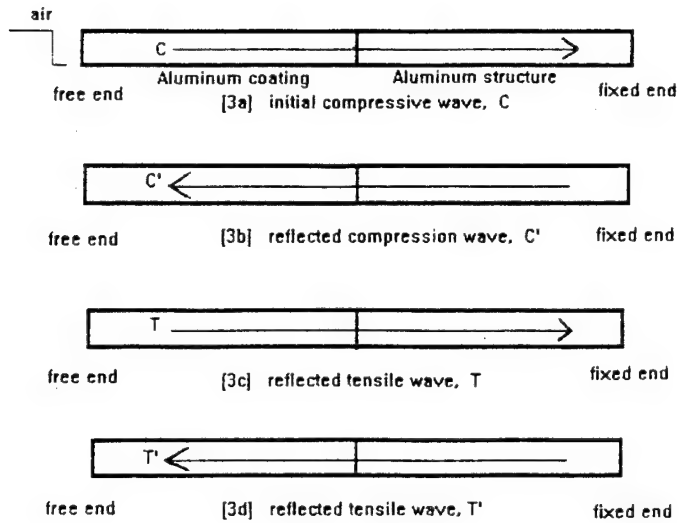
**Figure 3. Homogeneous aluminum system ($E_c/E_s = 1$) subjected to unit step pressure wave**



Homogeneous: Ec/Es = 1
3a = initial compressive wave
3b = reflected compressive wave
3c = initial tensile wave
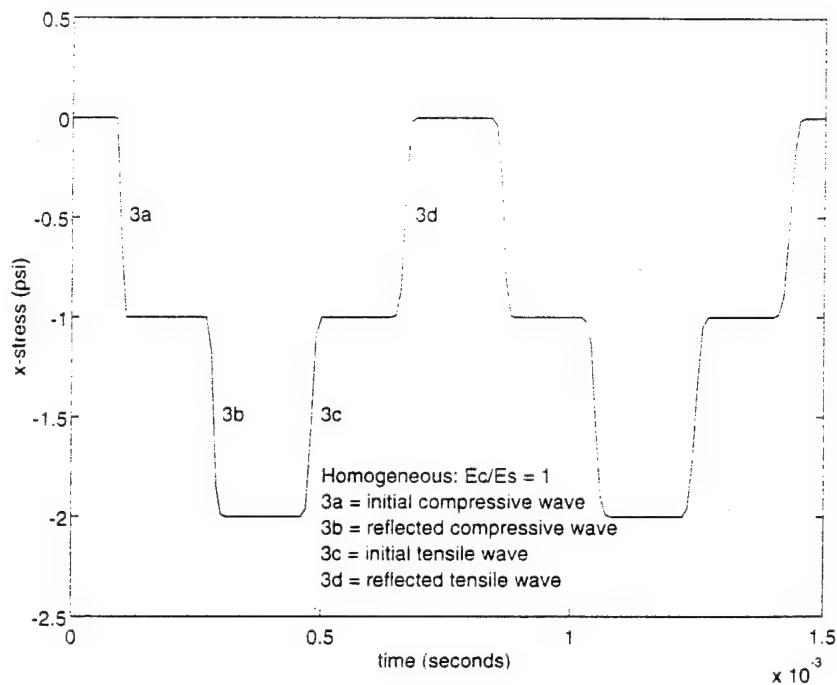3d = reflected tensile wave

**Figure 4.  Stress history for homogeneous system at a point near the interface on the structure**

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(Figure 5a). This transmitted wave will travel at a speed more than three times faster than the reflected wave due to the larger characteristic impedance of the structure. The transmitted wave is reflected back into the structure at the fixed end as a compressive wave of the same magnitude (Figure 5b). This reflected wave strikes the interface as it propagates back towards the coating. Since it encounters a less stiff material, the stress wave is reflected back into the structure as a tensile wave, and it thereby relieves some of the net compressive stress in the structure. Meanwhile a net compressive stress wave of very small magnitude is transmitted into the coating (Figure 5c). The tensile wave is reflected as reflected as a tensile wave at the fixed boundary (Figure 5d). The compressive wave initially reflected at the interface into the coating reaches the free end and is re-reflected as a tensile wave (Figure 5e). It is this wave which relieves the largest portion of the net compressive stress bringing the system back to a near-zero stress state. Figure 6 summarizes the above events at a point on the structure.

The same behavior is observed for systems with coatings 100 and 1000 times less stiff. The time of a complete cycle, specifically the time from compression to release, increases with decreasing coating stiffness. This is because a less stiff coating has a smaller acoustic velocity. The number of alternating small magnitude compression and tension cycles in the structure increases as well. The net compressive stress values remain constant, but the magnitude of the transient spikes increases with decreasing coating stiffness (Figure 7).

### c. *Coating Stiffer than Structure*

If the stiffness of the coating is increased by factor of 10 (i.e., $E_c/E_s = 10$), the compressive wave from the air to the system has a different dynamic response. Since the structure is less stiff than the coating, the reflected wave at the interface is tensile while the transmitted wave is

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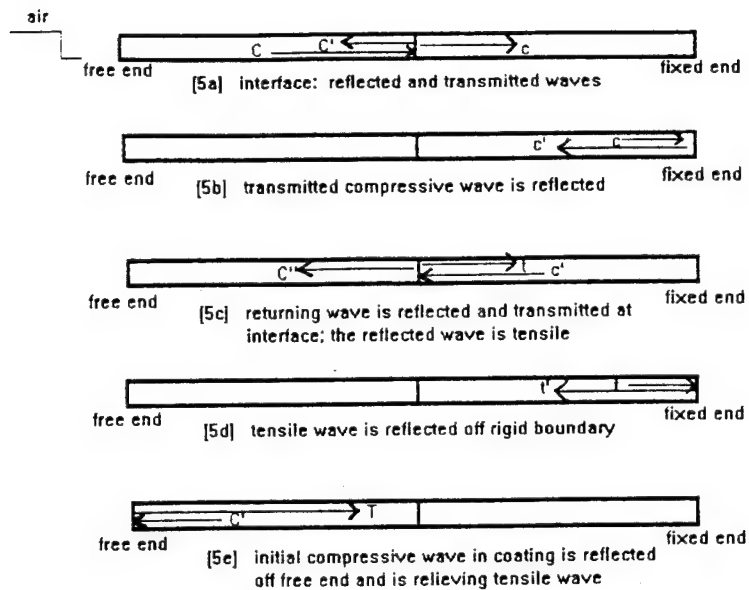
Figure 5. System with aluminum coating 10 times less stiff
($E_c/E_s$ = 0.1) subjected to unit step pressure wave

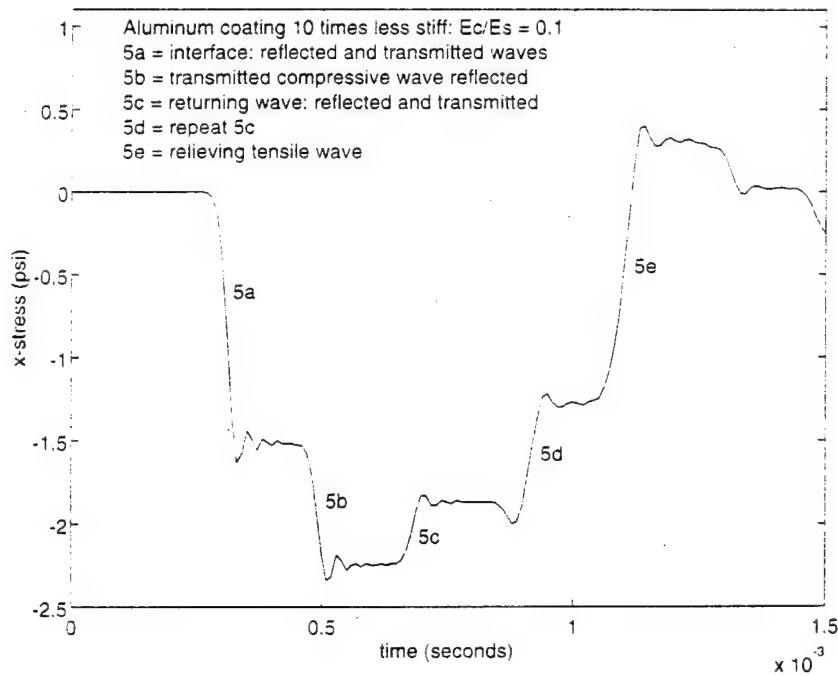

Figure 6. Stress history at a point near interface on the
structure with coating 10 times less stiff ($E_c/E_s$ = 0.1)

15

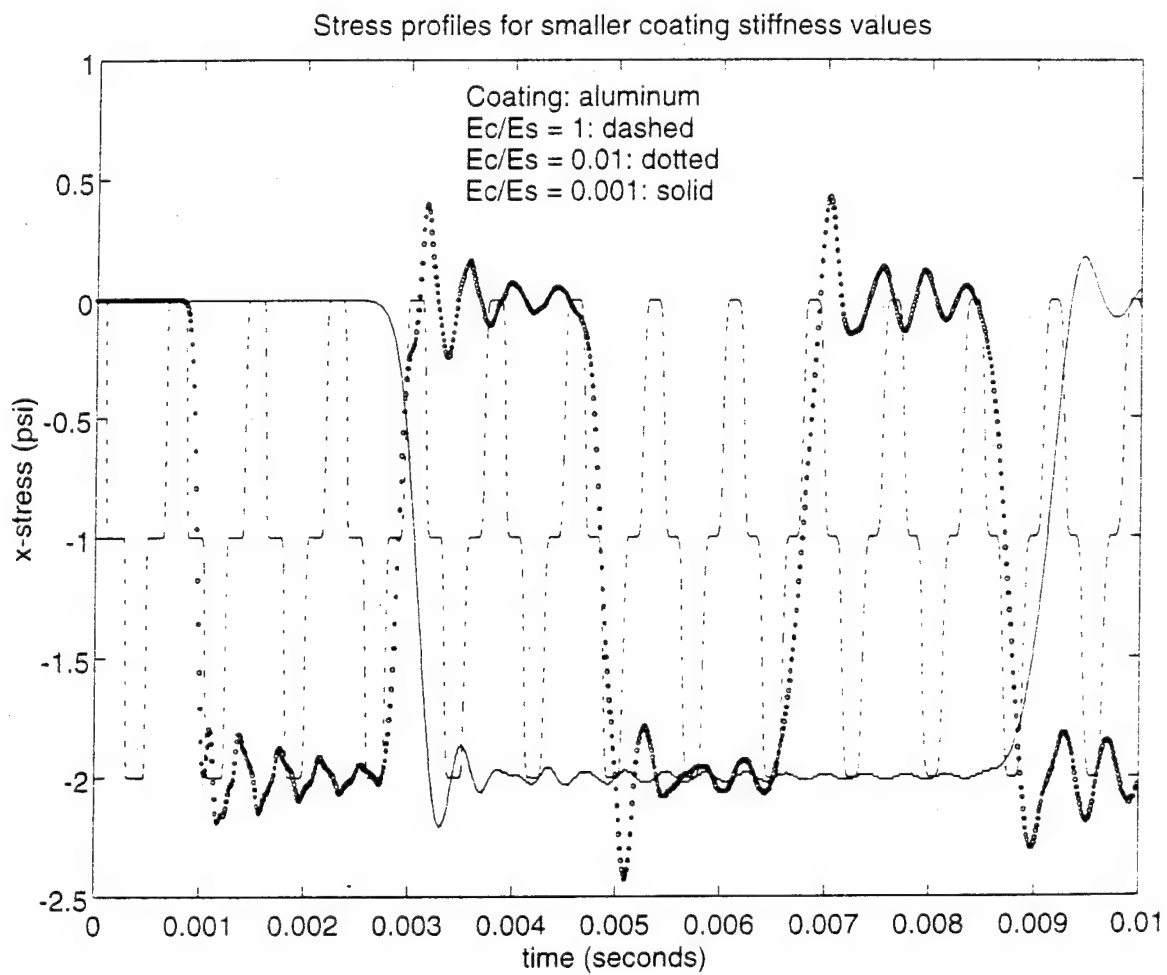Stress profiles for smaller coating stiffness values

Figure 7. Stress profiles at a point near the interface on
the structure for aluminum coated structures: homogeneous,
$E_c/E_s = 0.01$, $E_c/E_s = 0.001$

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compressive (Figure 8a). The tensile reflected wave travels at roughly three times faster than that in the coating due to a larger characteristic impedance. This wave is reflected at the free end as a compressive wave of equal magnitude. This second compressive wave interacts at the interface producing a weaker reflected tensile wave and a weaker net compressive wave into the structure (Figure 8b). The process repeats itself with an even weaker net compressive wave into the structure (Figure 8c). Meanwhile the first compressive wave transmitted into the structure is reflected at the fixed boundary as a compressive wave thereby increasing the net compressive stress of the structure (Figure 8d).

The competing effects of alternating compression and tension are observed until the compressive wave is relieved completely. (Figure 8e). Figure 9 depicts the response of a system with an aluminum coating 10 times stiffer.

There is little difference in the stress resulting from an aluminum stiffness increased by a factor of 100 or even 1000 (Figure 10). The stiffer coating example is used as a means of understanding the mechanics of stress wave behavior in a solid. In general, the coating will be less stiff than the structure it is designed to protect.

## 2. Viscoelastic Coating

Up to this point, the coating and structure have had similar characteristics; only the stiffness of the coating had been varied. A greater degree of complexity is added when the coating material type is changed altogether.

A more realistic structural coating entails substituting the elastic aluminum coating with viscoelastic tread stock rubber. The unit step pressure wave transmitted to the system will now first transit the viscoelastic coating, which will have a significant impact on the wave characteristics when it reaches the structure. Furthermore, the response may not be as predictable.

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air

free end     [8a]   initial transmitted compressive wave     fixed end

free end     [8b]   second transmitted compressive wave
from first reflected tensile wave     fixed end

free end     [8c]   third transmitted compressive wave
from second reflected tensile wave     fixed end

free end     [8d]   initial transmitted compressive wave
reflected from the fixed boundary     fixed end

free end     [8e]   final wave in a series of relieving tensile
waves rendering zero-stress state system     fixed end

**Figure 8. System with aluminum coating 10 times stiffer ($E_c/E_s = 10$) subjected to unit step pressure wave**

Aluminum coating 10 times stiffer: Ec/Es = 10
8a = initial transmitted compressive wave
8b = second transmitted compressive wave
8c = third transmitted compressive wave
8d = initial compressive wave reflected from fixed end
8e = final relieving tensile wave

**Figure 9. Stress history at a point near the interface on structure with coating 10 times stiffer ($E_c/E_s = 10$)**

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Figure 10. Stress profile at a point near the interface on
the structure for aluminum coated structures: homogeneous,
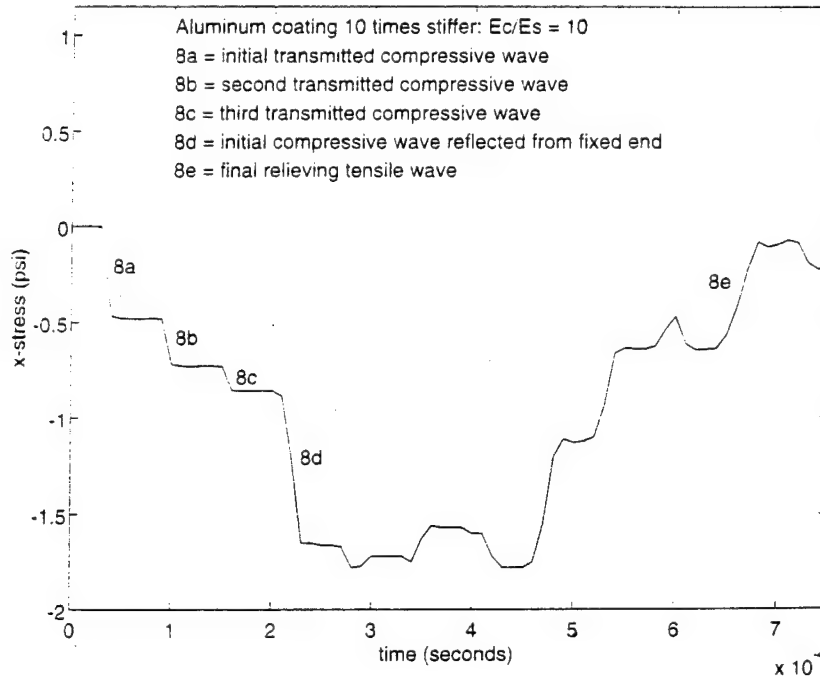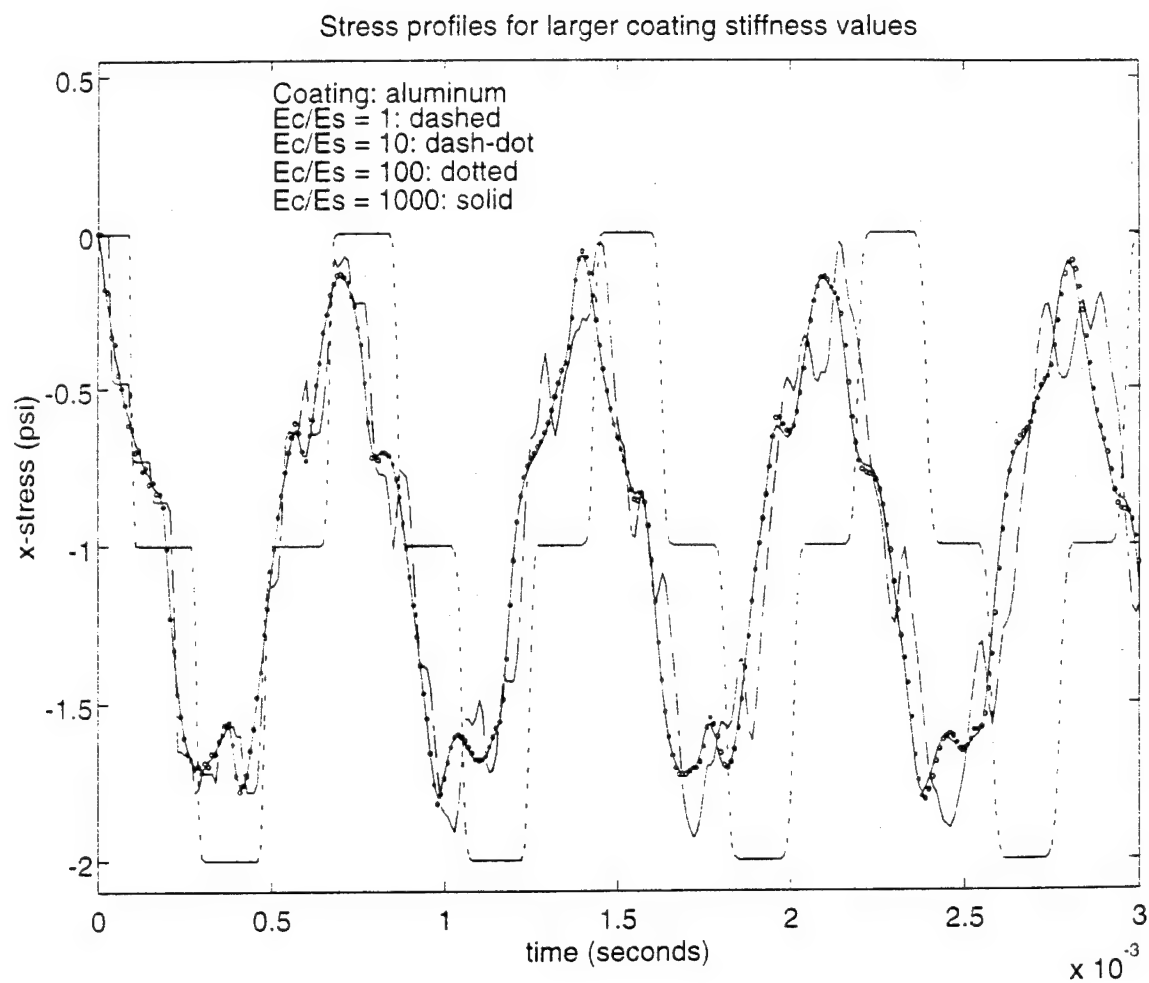$E_c/E_s = 10$, $E_c/E_s = 100$, $E_c/E_s = 1000$

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Experimental research conducted by Kolsky on wave propagation in viscoelastic solids [Ref. 12] concludes that the phase velocity and attenuation of stress waves increase as a function of distance in the direction of wave travel. Thus a wave propagating through the viscoelastic solid will both broaden and become asymmetrical in shape. Since the focus of this study is the dynamic response of the structure, the specific details of the wave propagation mechanics in the coating will not be covered.

An observation of the stress response shows that the tread stock rubber coating produces larger magnitudes of stress in the aluminum structure when compared to that in the homogeneous structure (Figure 11). The initial compressive wave is alternately reflected at the fixed end and re-reflected at the interface in a series of compressive and tensile waves. The stress wave alternates compressive and zero-stress states much slower than the aluminum coated models causing the structure to remain in a compressive stress state for a longer period of time.

Suppose the properties of tread stock rubber remain constant only the coating shear modulus is increased by a factor of 10. Both the average stress wave magnitude and periodicity decrease, but the stress at a point in the rubber coated structure exceeds the stress in the homogeneous aluminum structure (Figure 12). In other words, the dynamic response is adverse. If the rubber shear modulus is increased by a factor of 100, the dynamic response of the rubber coated structure is favorable when compared to the homogeneous aluminum response (Figure 13); the stress state is smaller.

A "threshold" value of a shear modulus for the rubber coating is apparent between 958 psi and 9580 psi. Further investigation reveals that a rubber shear modulus of 6000 psi and greater will render a favorable dynamic response for the rubber coated, one-dimensional, air-bounded model (Figure 14).
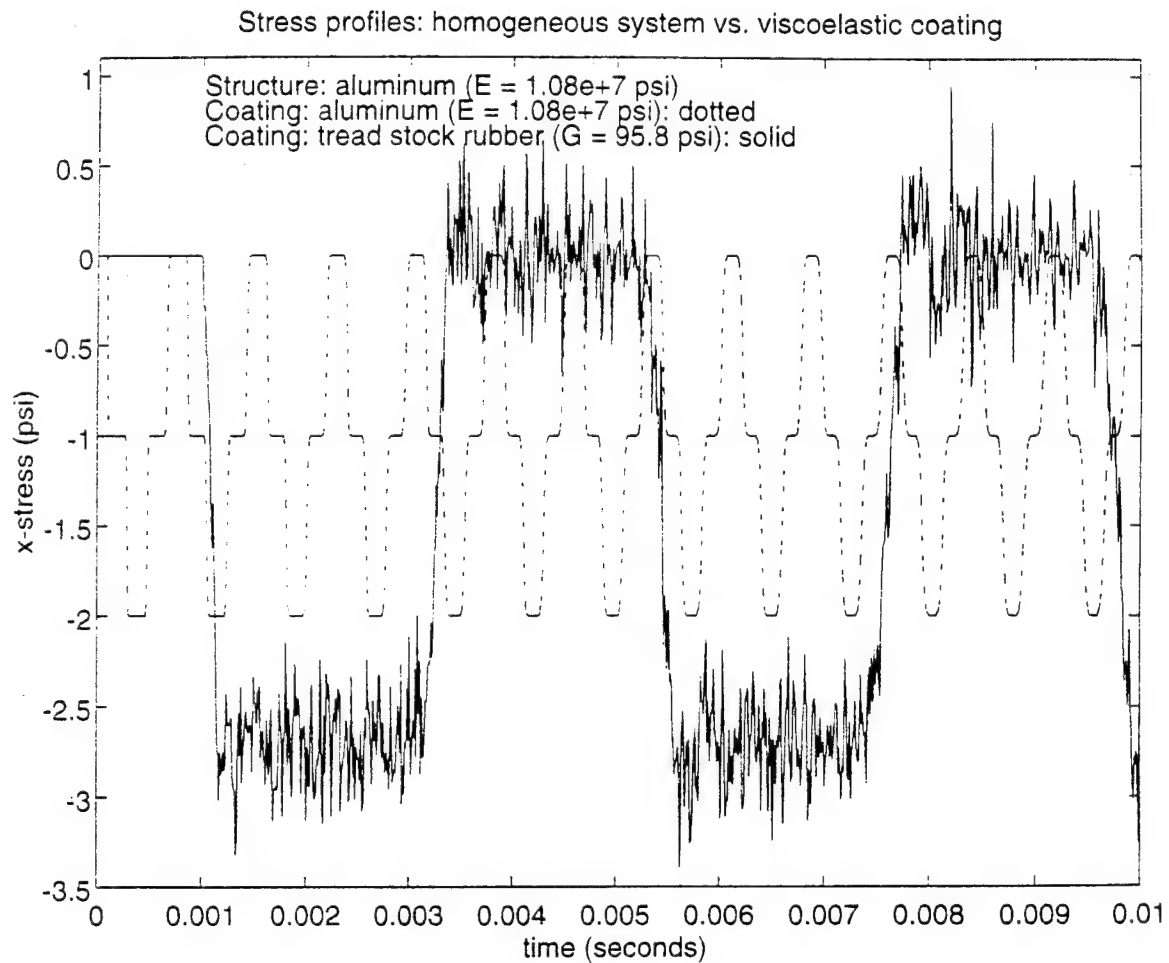
20

Stress profiles: homogeneous system vs. viscoelastic coating

**Figure 11. Stress profiles at a point near the interface on the structure comparing aluminum (E=1.08e+7 psi) and viscoelastic tread stock rubber (G=95.8 psi) coated systems subjected to unit step pressure wave**

**Figure 12. Comparing stress profiles at point near interface on structure for aluminum versus tread stock rubber coating (G = 958 psi)**
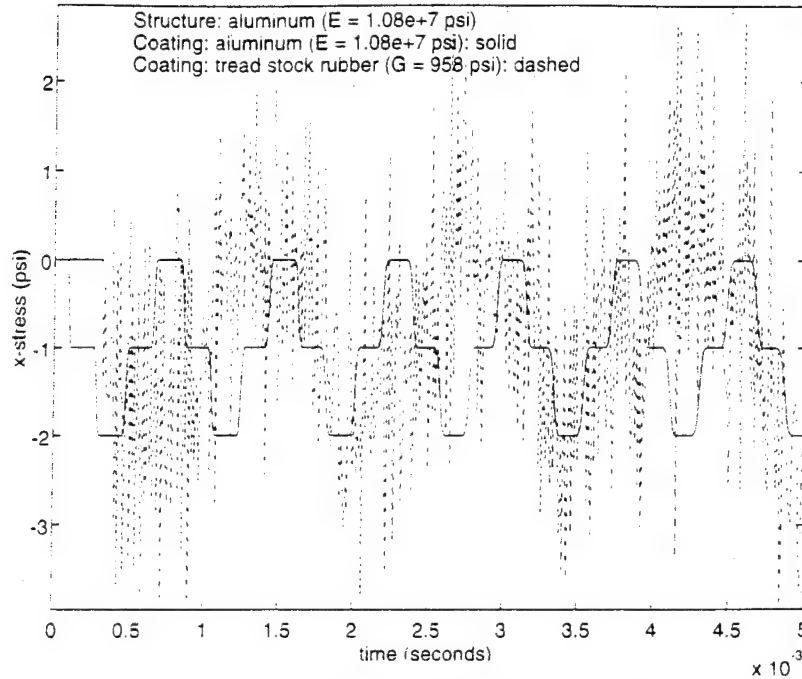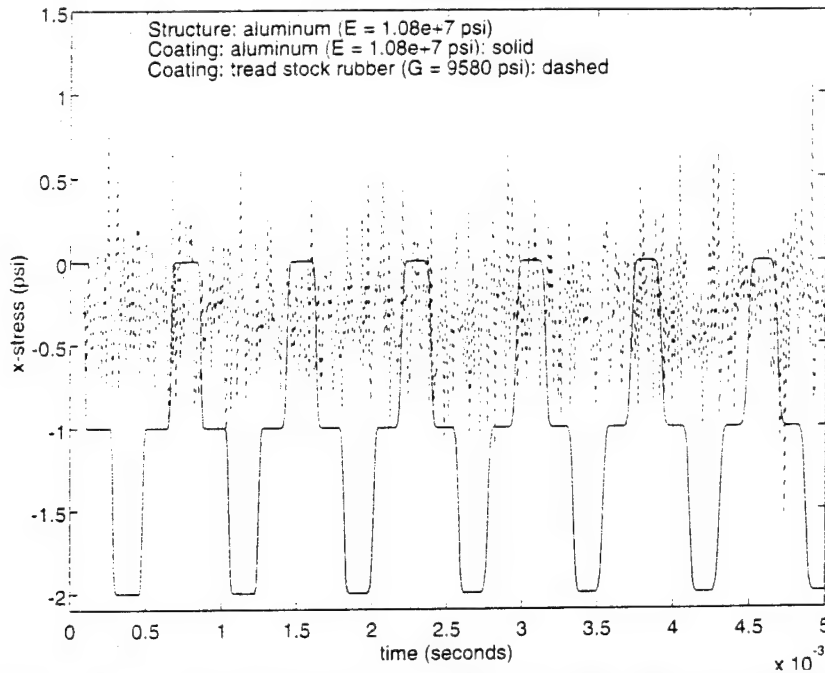


**Figure 13. Comparing stress profiles at point near interface on structure for aluminum versus tread stock rubber coating (G = 9580 psi)**
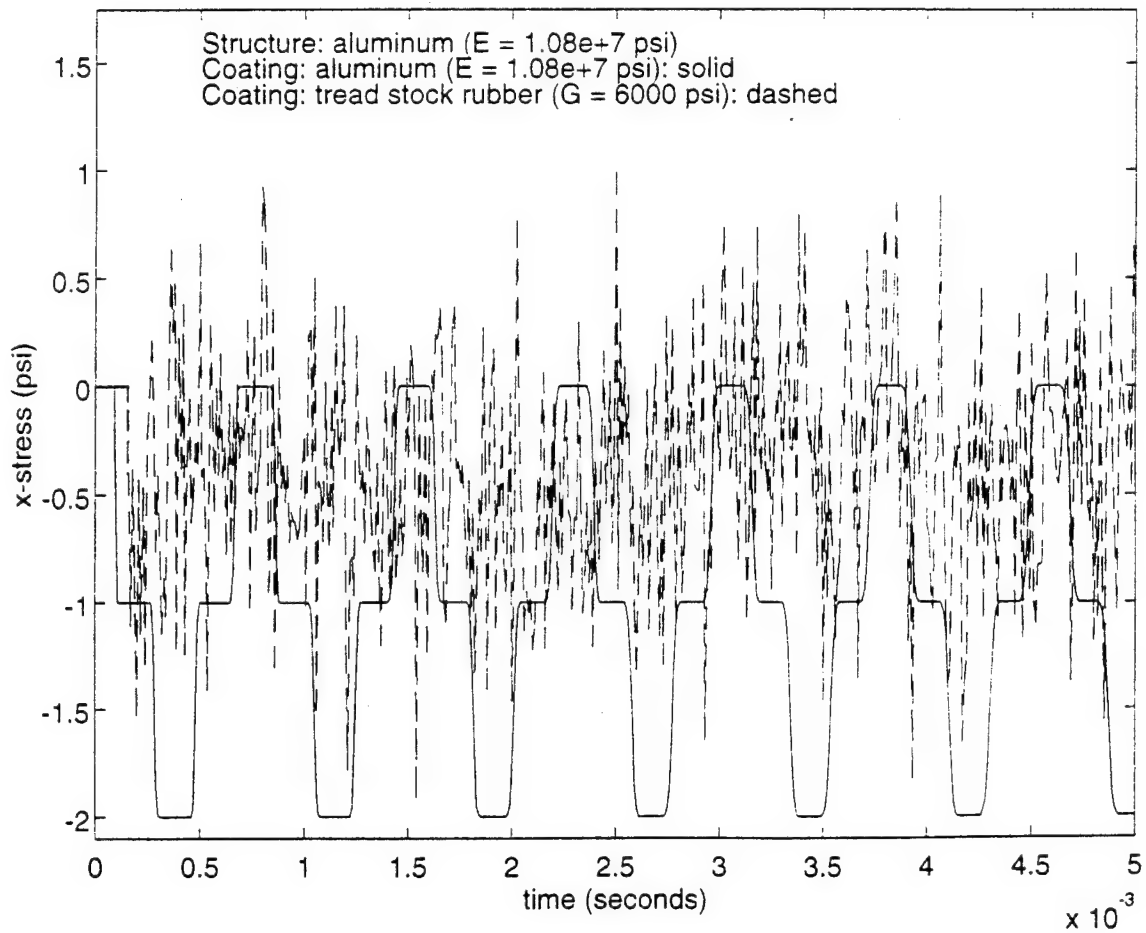
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**Figure 14. Comparing stress profiles at point near interface on structure for aluminum versus tread stock rubber coating (G = 6000 psi)**

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In general, the stress induced in the structure increases with less stiff viscoelastic coatings. The softer viscoelastic coating appears to inhibit stress energy release from the structure to the coating and to the medium. This excess energy is trapped within the structure. The result is a higher stress state, which elevates the structural material closer to yield stress limits. In more extreme instances, plastic deformation and failure of the structure ensues.

## B.    FREE END BOUNDED BY WATER

Previously, all the systems studied have had the free end of the system bounded by an air medium. The scope of the analysis now shifts to a more realistic application by observing a water-bounded system subjected to the same conditions previously prescribed.

The characteristic impedance of air is essentially negligible. Thus there is little wave energy transmission to the air from the system. A compressive wave interacting with the free end will be reflected as a tensile wave of nearly the same magnitude and vice-versa. The introduction of a water medium at the free end alters the dynamic response of the system. The interaction of the stress wave with the water at the free end will not be as ideal.

The water is a material with a characteristic impedance approximately 3600 times that of air. Stress wave energy will be more readily transmitted from both the structure and the coating to the water medium.

As shown in the following derivation, the velocity at any point on the structure is proportional to the stress. As a result, the nodal velocity at a point on the structure is used to compare the dynamic response of the system bounded by water for various coatings.

For a one-dimensional element on the structure, recall from (12) that the total displacement is expressed as:

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$$u = F(c_o t - x) + f(c_o t + x)$$

If the wave is travelling in the direction of decreasing $x$,

$$u = F(c_o t - x) \qquad \qquad \textbf{(17)}$$

then differentiate both sides with respect to displacement, $x$, to get:

$$\frac{\partial u}{\partial x} = -F'(c_o t - x) \qquad \qquad \textbf{(18)}$$

If (17) is differentiated with respect to time, $t$:

$$\frac{\partial u}{\partial t} = c_o F'(c_o t - x) \qquad \qquad \textbf{(19)}$$

from (18) and (19) above, the following expression is derived:

$$\frac{\partial u}{\partial t} = -c_o \frac{\partial u}{\partial x} \qquad \qquad \textbf{(20)}$$

and, finally, from equation of equilibrium, the result is:

$$\sigma_{xx} = -\left(\frac{E}{c_o}\right)\frac{\partial u}{\partial t} = -\rho c_o \frac{\partial u}{\partial t} \qquad \qquad \textbf{(21)}$$

This equation gives the relationship between the stress at any point on the structure and the particle velocity with the characteristic impedance as the proportionality constant. Henceforth, the dynamic response of the water-bounded systems will be given in terms of the nodal velocity.

## 1. Elastic Coating

Consider the homogeneous system, that is one using identical material for the coating and structure, subjected to a unit step pressure wave at the free end. The velocity response of a node on the structure side of the interface resembles that of the homogeneous system exposed to air. However, even though the two velocity profiles have identical time periods, the velocity of the water-bounded system decays

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to zero as time elapses (Figure 15). A similar response results regardless of the coating stiffness (Figures 16, 17, 18, 19). The air-bounded system cycles at the same amplitude throughout the duration of the pressure pulse. The water damps out a portion of the stress wave energy; this allows the system to return to a lower energy state as time goes on.

The stiffness of the coating influences the nodal velocity of the structure. This velocity is indicative of the stress state in the structure; a higher stress state is characterized by a higher velocity. The peak nodal velocity increases while the period between successive peaks decreases with decreasing aluminum coating stiffness (Figures 20, 21). The softer aluminum coating inhibits release of stress wave energy from the structure to the surrounding water medium. The excess energy is manifested in the form of higher nodal velocities and stresses in the underlying structure. Therefore, a higher stress state accompanies a less stiff elastoplastic coating.

### 2. Viscoelastic Coating

Comparing the velocity profile of the air-bounded to the water-bounded tread stock rubber coated system, there is a distinct difference in the velocity of a structural node just inside the coating/structure interface. The nodal velocity of the air-bounded system is greater. Thus the air-bounded system has a higher stress magnitude regardless of the rubber stiffness (Figure 22). This higher stress state is indicative of less stress wave energy being released to the air when compared to the energy dissipation to the water.

As previously discussed, a point on the structure of an aluminum coated system has a velocity profile decreasing with time. On the other hand, the response of the rubber coated system is not as smooth or predictable. The viscoelastic coating yields an erratic nodal velocity characterized by alternating peaks and nadirs of unequal magnitude (Figure 23).
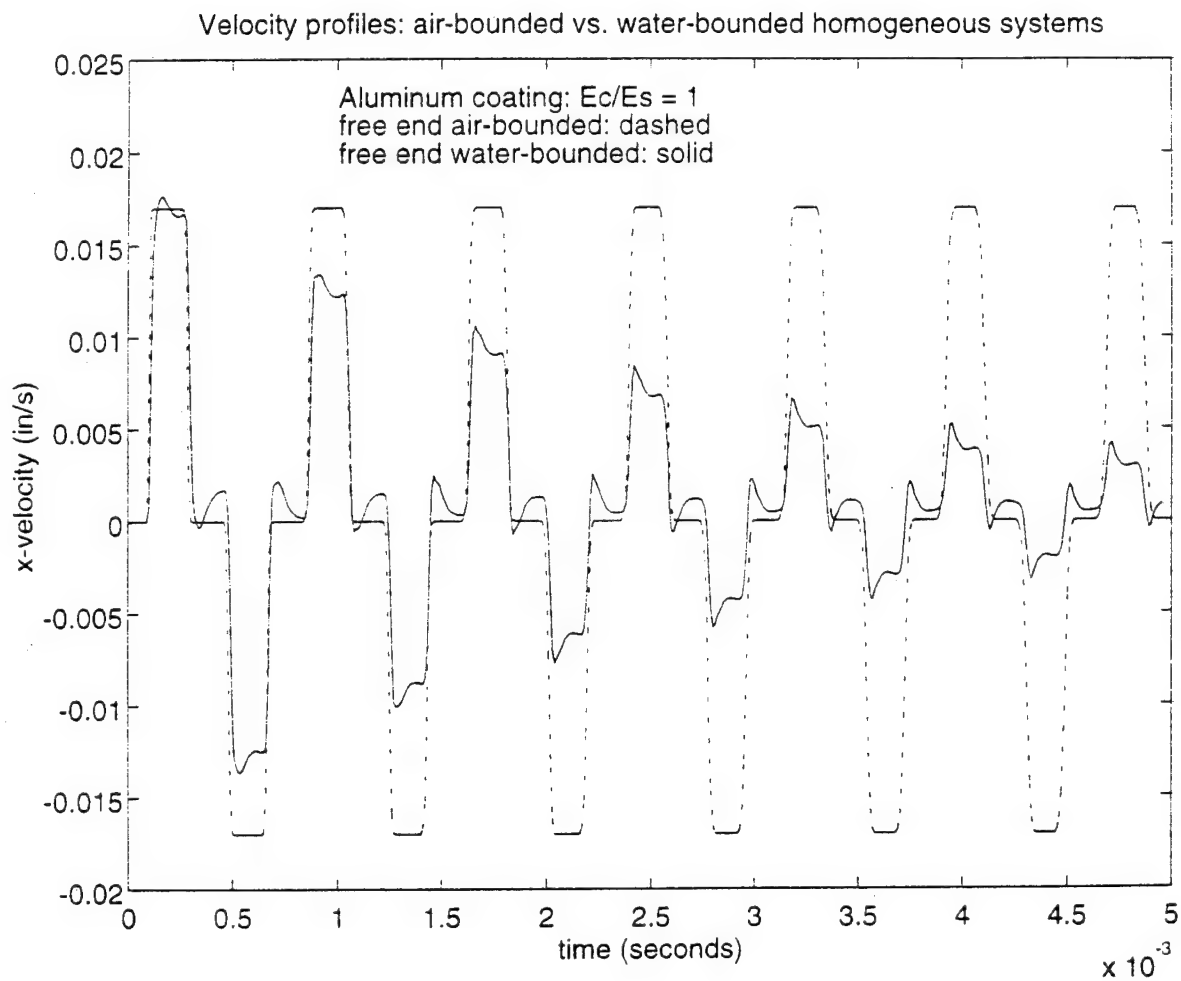
26

**Figure 15.** Comparing velocity profiles at a point near the interface on structure for air-bounded versus water-bounded homogeneous systems

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**Figure 16. Comparing velocity profiles at a point near interface on structure for air-bounded vs. water-bounded coatings ($E_c/E_s = 0.1$)**



**Figure 17. Comparing velocity profiles at a point near interface on structure for air-bounded vs. water-bounded coatings ($E_c/E_s = 0.01$)**
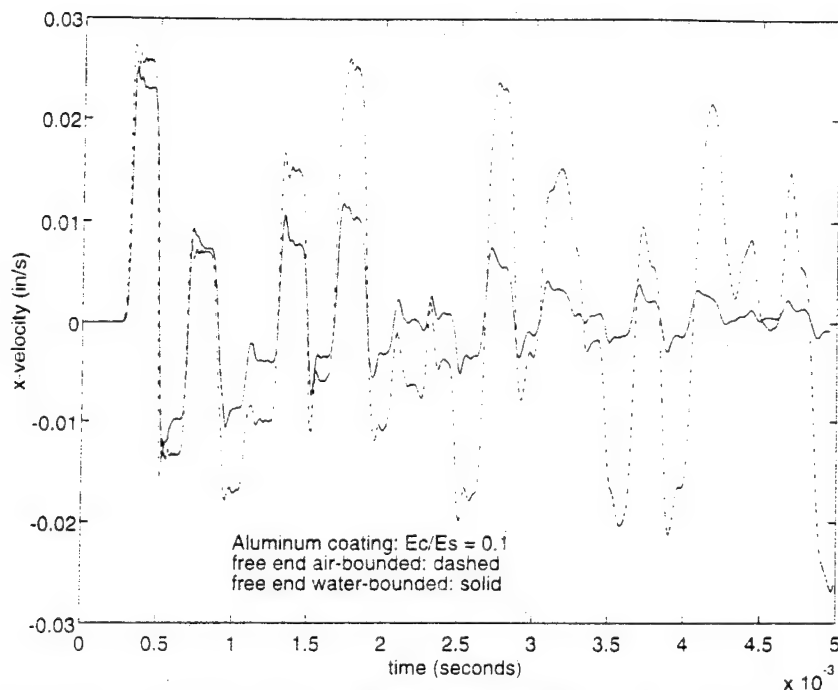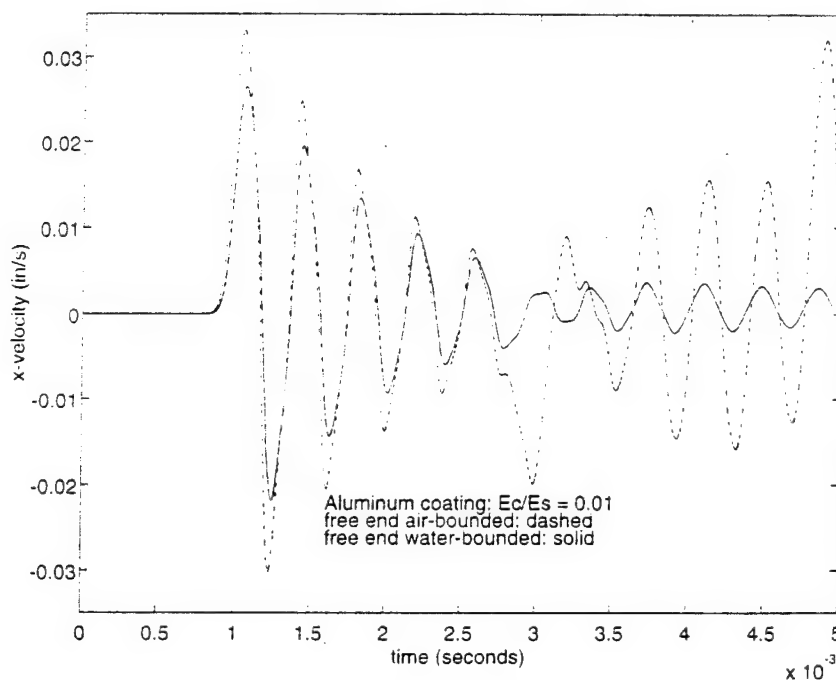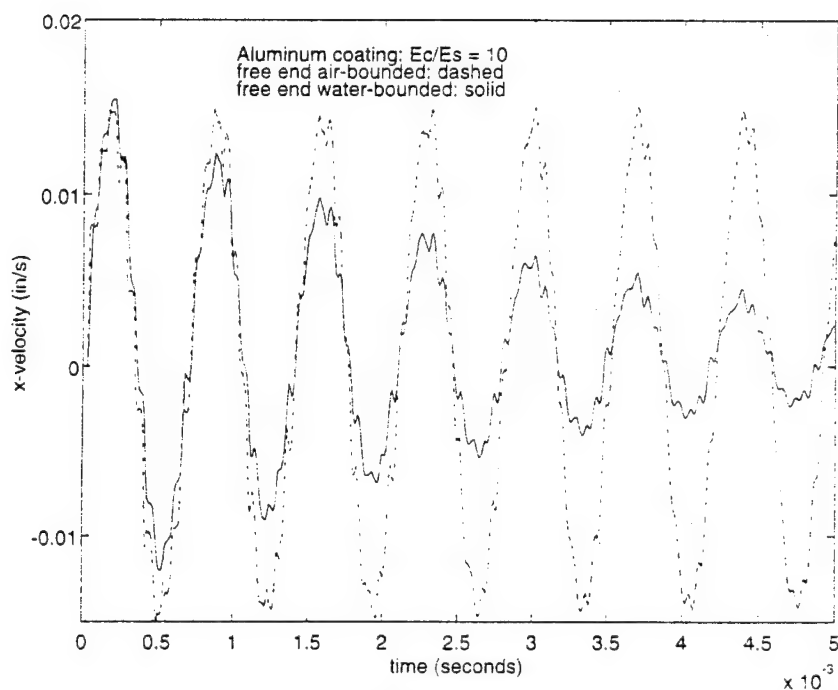
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**Figure 18. Comparing velocity profiles at a point near interface on structure for air-bounded vs. water-bounded coatings ($E_c/E_s = 10$)**
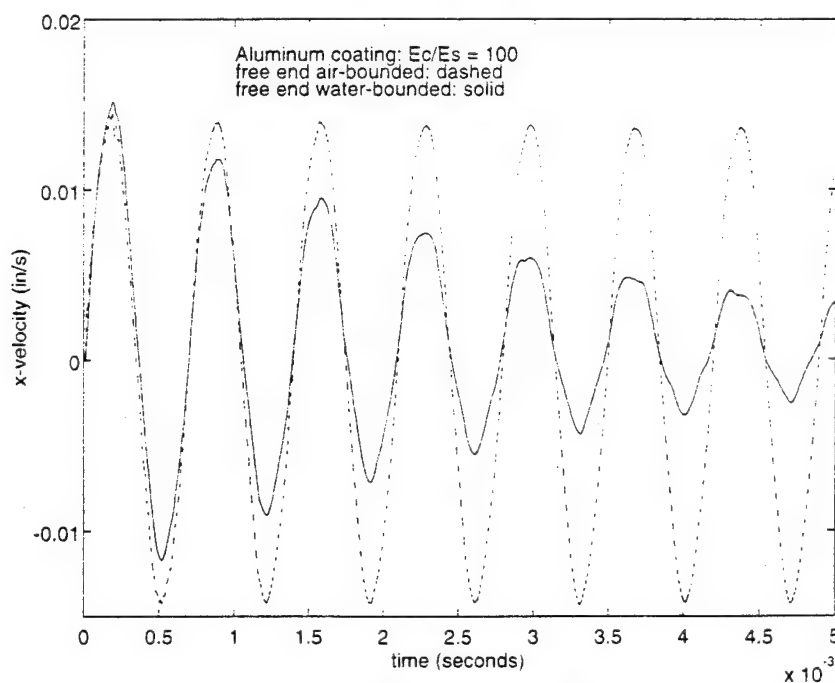


**Figure 19. Comparing velocity profiles at a point near interface on structure for air-bounded versus water-bounded coatings ($E_c/E_s = 100$)**

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**Figure 20.** Comparing velocity profiles at a point near the interface on the structure for water-bounded systems with less stiff aluminum coatings: homogeneous, $E_c/E_s = 0.1$, $E_c/E_s = 0.01$

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Velocity profiles: water-bounded varying coating stiffnesses

Coating: aluminum
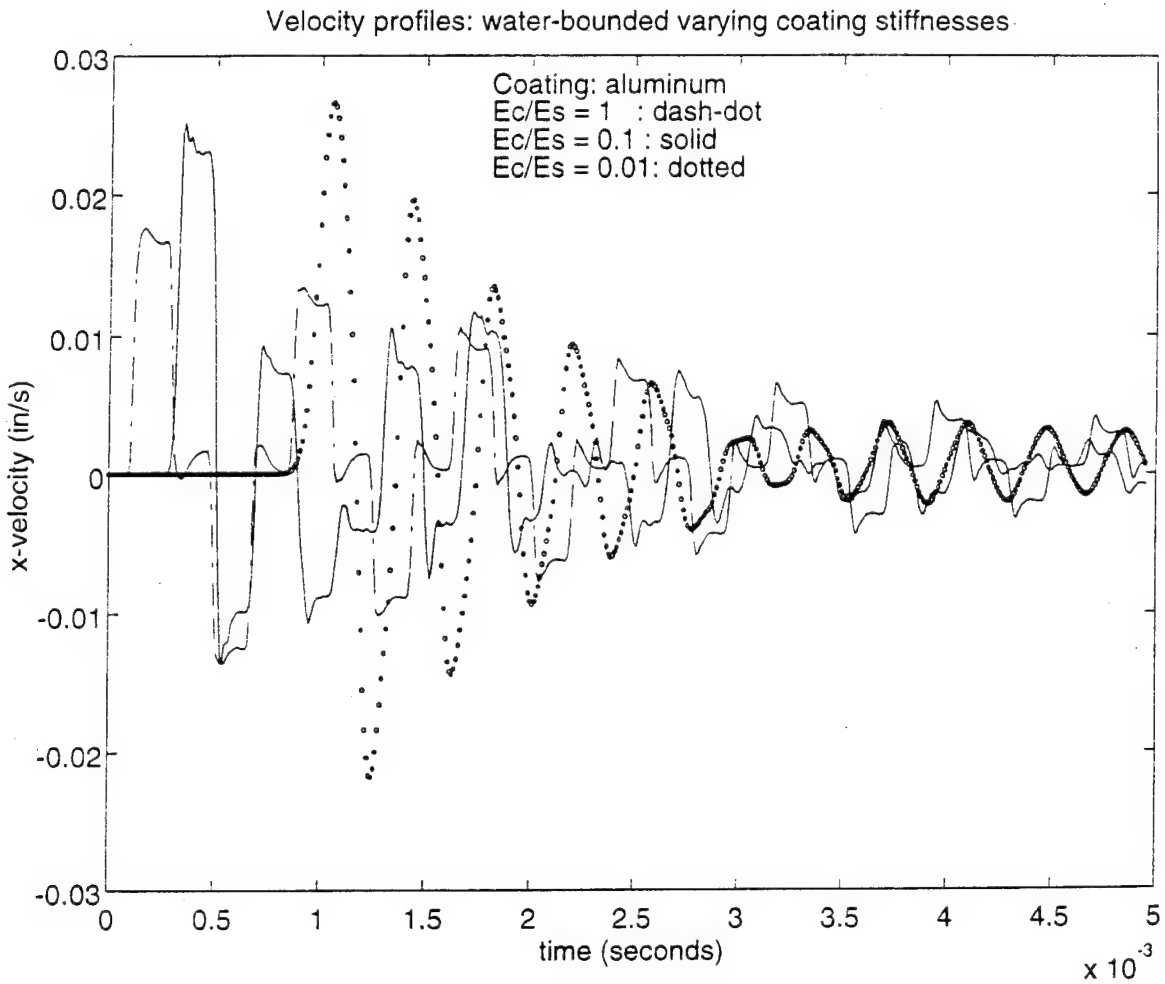Ec/Es = 1  : dash-dot
Ec/Es = 10 : solid
Ec/Es = 100: dotted
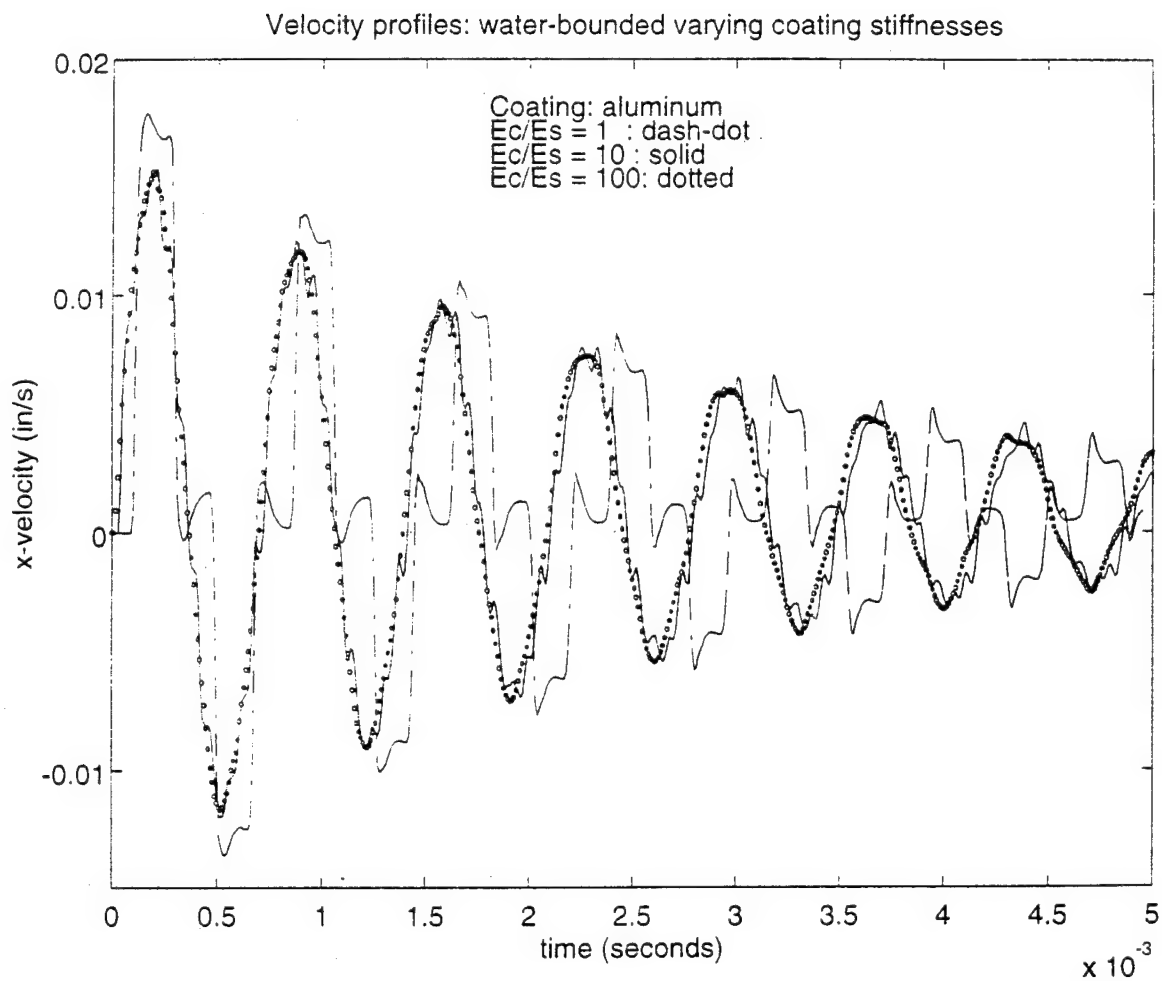
Figure 21. Comparing velocity profiles at a point near the interface on the structure for water-bounded systems with more stiff aluminum coatings: homogeneous, $E_c/E_s = 10$, $E_c/E_s = 100$

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Velocity profiles: water vs. air-bounded viscoelastic coating

Coating: tread stock rubber (G = 95.8 psi)
free end air-bounded: dashed
free end water-bounded: solid

Figure 22. Comparing velocity profiles at a point near
interface on structure for air-bounded vs. water-bounded
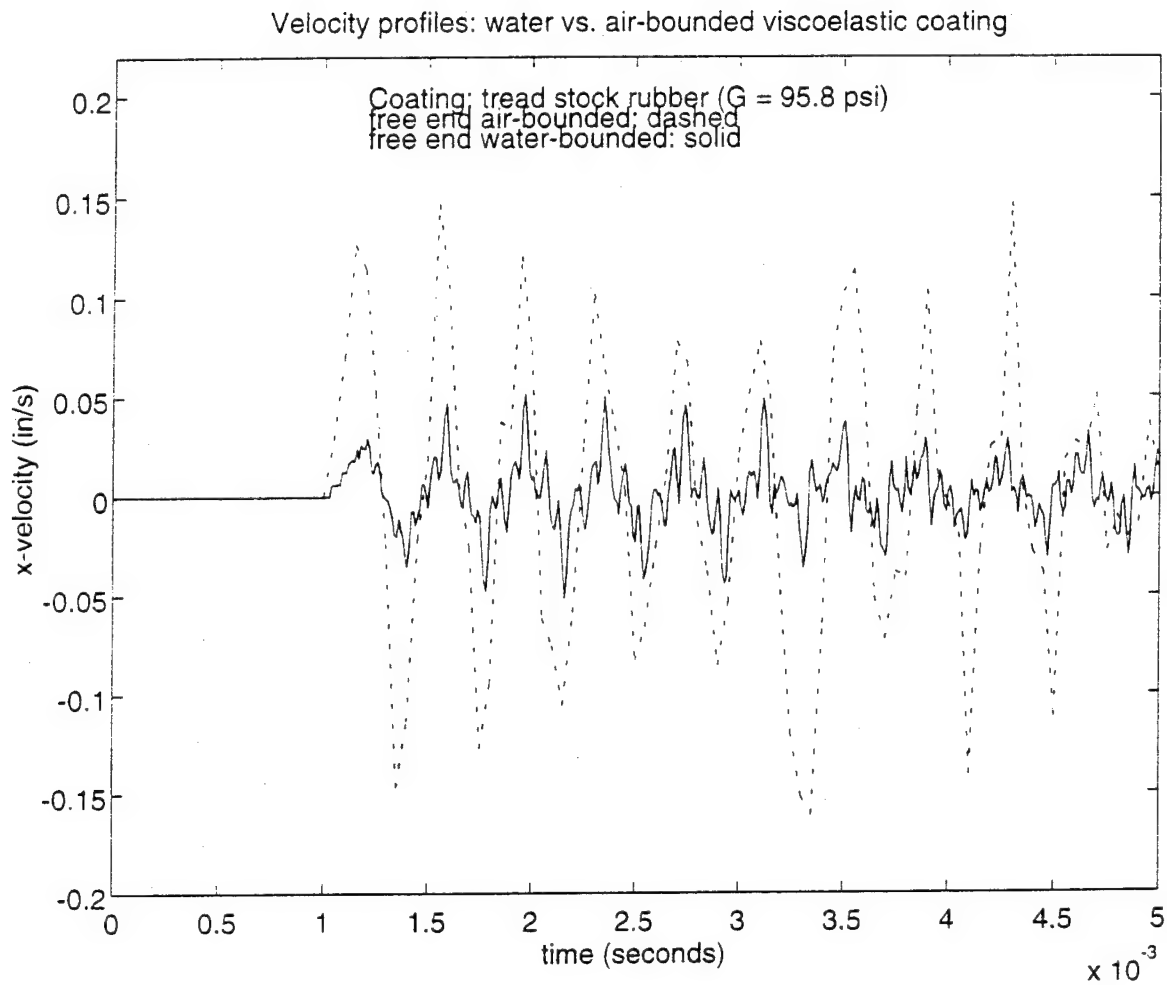systems with tread stock rubber (G = 95.8 psi) coating

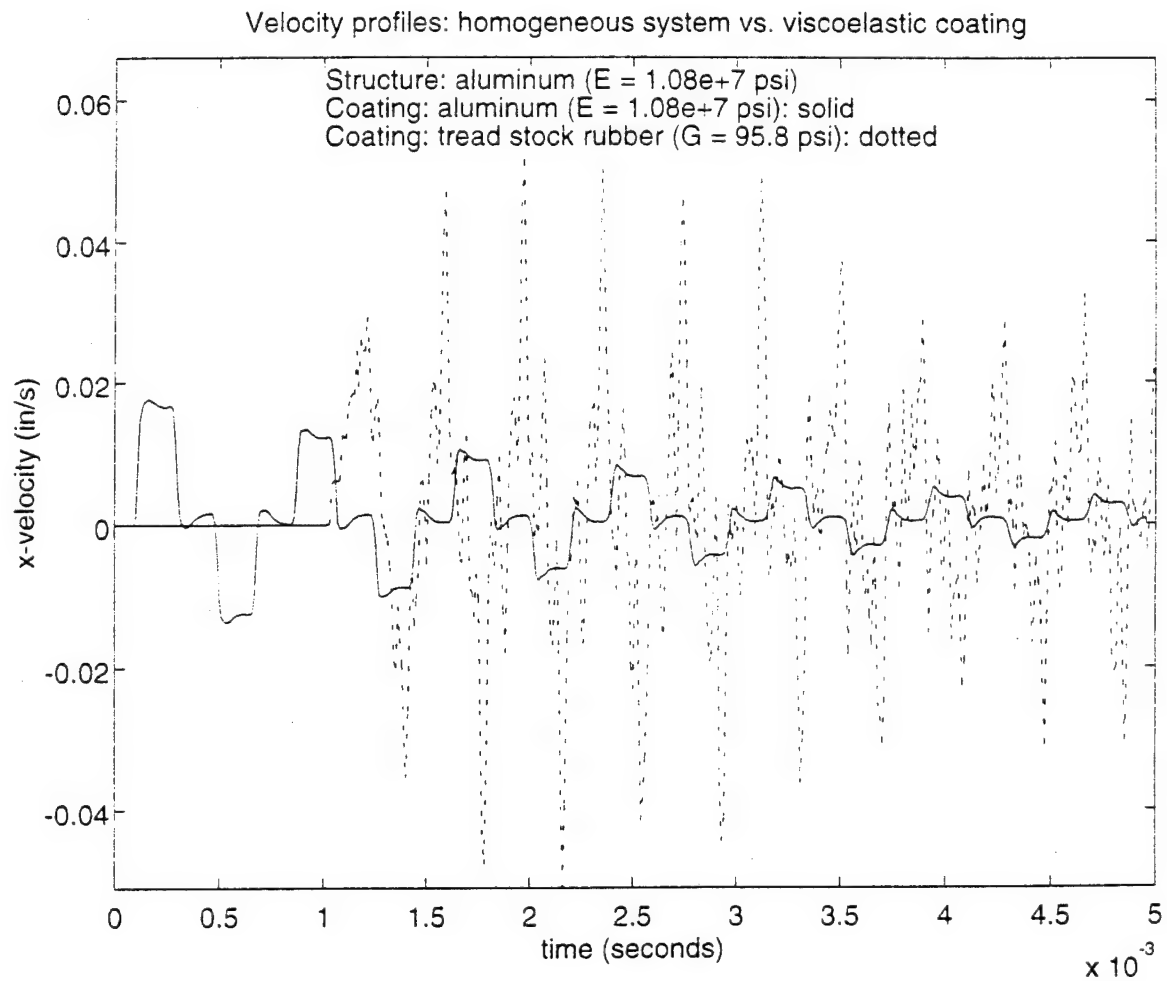32

Velocity profiles: homogeneous system vs. viscoelastic coating



Figure 23. Comparing velocity profiles at a point near
interface on structure for water-bounded aluminum versus
tread stock rubber coated (G = 95.8 psi) systems

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Increasing the shear modulus of the tread stock rubber by a factor of 10 results in a smaller, more refined velocity profile. However, a higher nodal velocity in the structure indicates that this dynamic response is worse than the homogeneous system response (Figure 24). A more favorable dynamic response results when the rubber shear modulus is increased by a factor of 100; the nodal velocity is smaller than the nodal velocity of the homogeneous system (Figure 25).

Like the air-bounded model, a threshold coating value for a one-dimensional, water-bounded system exists at a rubber shear modulus between 958 psi and 9580 psi. Further investigation reveals that a shear modulus of 6000 psi and greater will result in a more favorable dynamic response (Figure 26).

The nodal velocity of a point in the structure increases with a decrease in the rubber stiffness regardless of the bounding medium. The softer coating serves to trap the stress wave energy within the structure preventing energy dissipation from the structure to the water. This effect raises the overall stress state of the underlying structure closer to yield stress limits.

**Figure 24. Comparing velocity profiles at a point near interface on structure for water-bounded aluminum versus stiffer tread stock rubber (G = 958 psi) coating**



**Figure 25. Comparing velocity profiles at a point near interface on structure for water-bounded aluminum versus stiffer tread stock rubber (G = 9580 psi) coating**

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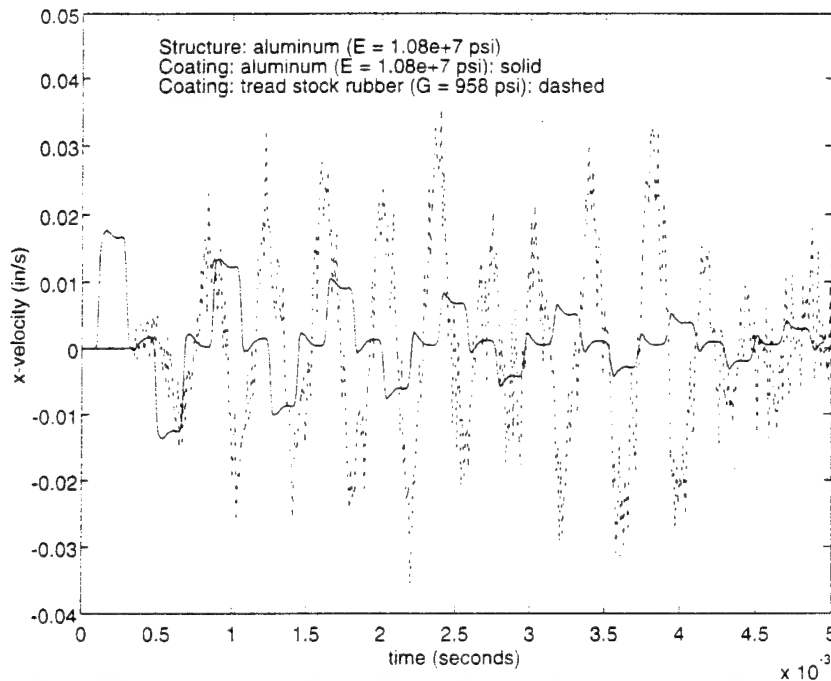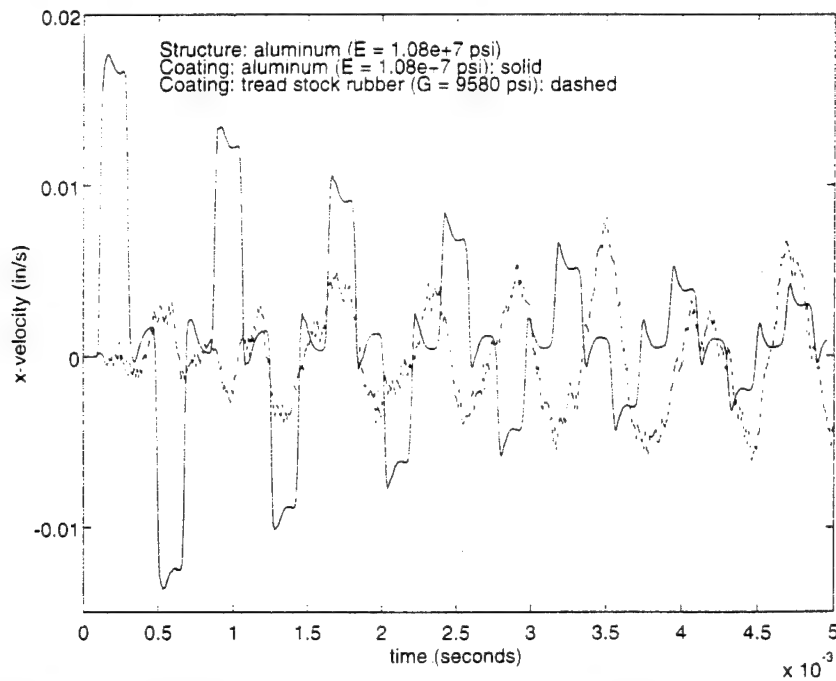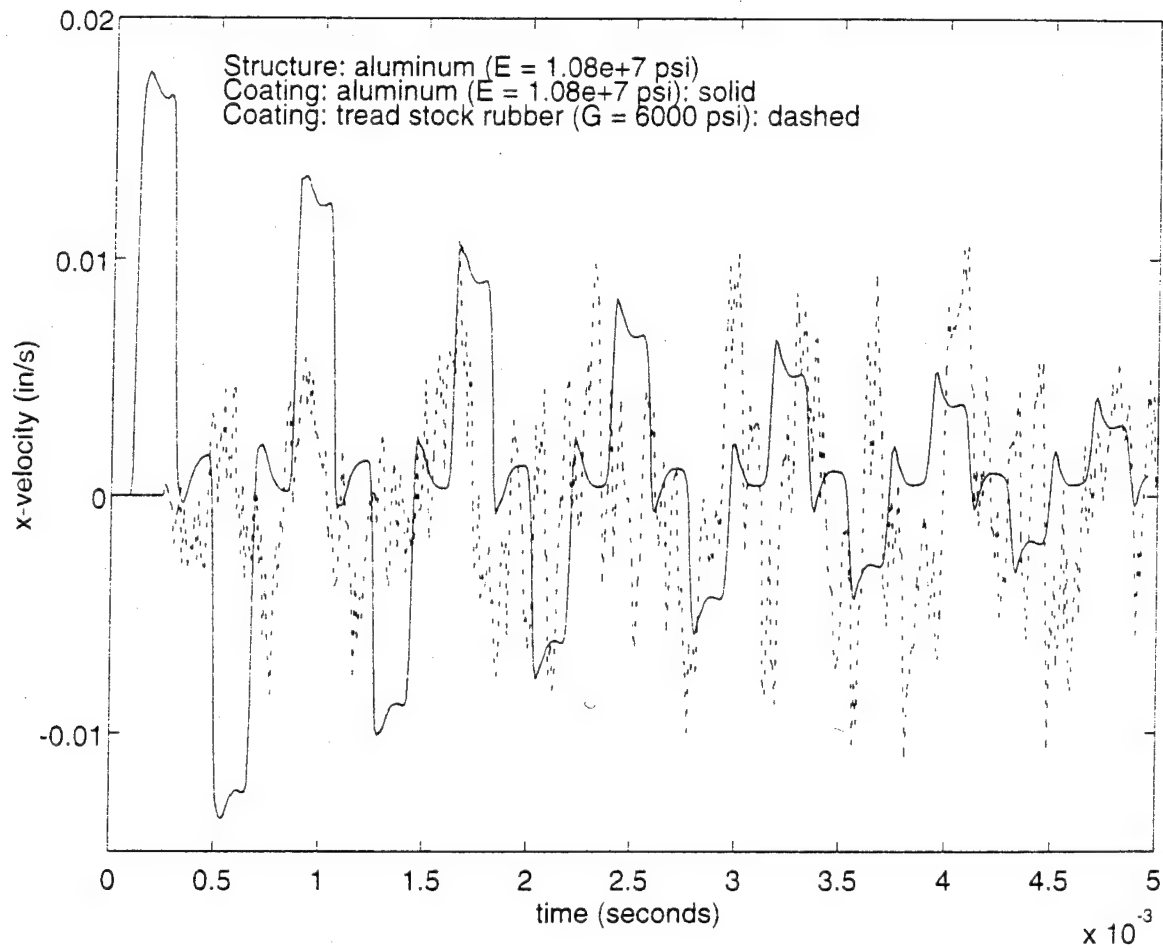Figure 26. Comparing velocity profiles at a point near
interface on structure for water-bounded aluminum versus
tread stock rubber (G = 6000 psi) coating

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## V. CONCLUSIONS

This parametric study analyzes the dynamic response of a simple, one-dimensional, coated structure subjected to a unit step pressure wave with the free end bounded by either air or water.

The air-bounded system response is characterized by alternating compression and release of the structure regardless of the nature of the coating material. This cyclical response is repeated throughout the duration of the external loading. The aluminum coating with reduced stiffness induces a higher stress magnitude in the underlying aluminum structure. Fluctuation of the stress wave is manifested as a series of compressive and tensile waves between the initial compression wave and the final relieving wave. The period between successive zero-stress states increases with decreasing elastoplastic coating stiffness leaving the structure in a higher net compressive stress state for a longer duration of time. The viscoelastic coating induces a series of erratic compressive and tensile waves in the structure. In general, a decreasing rubber stiffness caused an increasing stress magnitude. Moreover, decreasing the stiffness of the rubber made the stress response difficult to predict. Much of this ambiguity lies in the unresolved phenomenon of stress wave propagation in a viscoelastic solid.

The release of a portion of the stress wave energy is evident in the water-bounded system. For an elastoplastic coating, the stress response is identical to that of the air-bounded system in its shape and periodicity only the water acts to dampen the stress energy. This damping results in reduced magnitudes in successive stress wave peaks as a function of time. The same phenomenon is observed in systems having viscoelastic coatings. The nodal velocity of the aluminum structure increases with decreasing rubber stiffness. The velocity response is erratic suggesting that the softer

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viscoelastic coating alters the magnitude of the compressive wave. In other words, a softer viscoelastic coating prevents the release of stress wave energy to the water, and it may even contribute to increased stress magnitudes within the structure. This study warrants further investigation into a structure coated with a viscoelastic material.

The results of this study show that a threshold value for a shear modulus exists for a rubber coated model. A more favorable dynamic response accompanies a shear modulus above this value, while an adverse effect results below this value. A rubber shear modulus of 6000 psi and greater ensures a more favorable response for the present structure regardless of whether the system is exposed to either an air or a water medium. The threshold value may vary depending upon the geometry and material properties of both the coating and the structure.

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